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# Novel features of the energy–momentum tensor of a Casimir apparatus in a weak gravitational field

Giuseppe Bimonte<sup>1,2</sup>, Enrico Calloni<sup>1,2</sup>, Giampiero Esposito<sup>2</sup> and Luigi Rosa<sup>1,2</sup>

<sup>1</sup> Dipartimento di Scienze Fisiche, Complesso Universitario di Monte S. Angelo Via Cintia, Edificio 6, 80126 Napoli, Italy

<sup>2</sup> INFN, Sezione di Napoli, Complesso Universitario di Monte S. Angelo Via Cintia, Edificio 6, 80126 Napoli, Italy

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## Abstract

The influence of the gravity acceleration on the regularized energy–momentum tensor of the quantized electromagnetic field between two plane parallel conducting plates is derived. A perturbative expansion, to first order in the constant acceleration parameter, of the Green functions involved and of the energy–momentum tensor is derived by means of the covariant geodesic point-splitting procedure. The energy–momentum tensor is covariantly conserved and satisfies the expected relation between gauge-breaking and ghost parts. Interestingly, a non-vanishing trace anomaly is obtained to first order in the constant acceleration.

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## 1. Introduction

An important property of quantum electrodynamics is that suitable differences of zero-point energies of the quantized electromagnetic field can be made finite and produce measurable effects such as the tiny attractive force among perfectly conducting parallel plates known as the Casimir effect [1]. This is a remarkable quantum-mechanical effect that makes itself manifest on a macroscopic scale. For perfect reflectors and metals the Casimir force can be attractive or repulsive, depending on the geometry of the cavity, whereas for dielectrics in the weak-reflector approximation it is always attractive, independently of the geometry [2]. The Casimir effect can be studied within the framework of boundary effects in quantum field theory, combined with the zeta-function regularization or Green-function methods, or in more physical terms, i.e. on considering van der Waals forces [3] or scattering problems [4]. Casimir energies are also relevant in the attempt of building a quantum theory of gravity and of the universe [5].

For these reasons, in [6] we evaluated the force produced by a weak gravitational field on a rigid Casimir cavity. Interestingly, the resulting force was found to have an opposite direction

to that of the gravitational acceleration; moreover, we found that the current experimental sensitivity of small force macroscopic detectors would make it possible, at least in principle, to measure such an effect [6]. In [6], calculations were based on simple assumptions and the result can be viewed as a reasonable *first order* generalization of  $T_{\mu\nu}$  from Minkowski to the curved spacetime. The present paper is devoted to a deeper understanding and to more systematic calculations of the interaction of a weak gravitational field with a Casimir cavity. In our approximation the former value of the force exerted by the field on the cavity is recovered to first order. But here we also find a trace anomaly for the regularized energy–momentum tensor.

We consider a plane-parallel Casimir cavity, made of ideal metallic plates, at rest in the gravitational field of the earth, with plates lying in a horizontal plane. We evaluate the influence of the gravity acceleration  $g$  on the Casimir cavity but neglect any variation of the gravity acceleration across the cavity, and therefore we do not consider the influence of tidal forces. The separation  $a$  between the plates is taken to be much smaller than the extension of the plates, so that edge effects can be neglected. We obtain a perturbative expansion of the energy–momentum tensor of the electromagnetic field inside the cavity, in terms of the small parameter  $\epsilon \equiv 2ga/c^2$ , to first order in  $\epsilon$ . For this purpose, we use a Fermi [7, 8] coordinates system  $(t, x, y, z)$  rigidly connected to the cavity. The construction of these coordinates involves only invariant quantities such as the observer’s proper time, geodesic distances from the worldline, and components of tensors with respect to a tetrad [8]. This feature makes it possible to obtain a clear identification of the various terms occurring in the metric. In our analysis we adopt the covariant point-splitting procedure [9, 10] to compute the perturbative expansion of the relevant Green functions. Gauge invariance plays a crucial role and we check it up to first order by means of the Ward identity.

In our notation, the  $z$ -axis coincides with the vertical upwards direction, while the  $(x, y)$  coordinates span the plates, whose equations are  $z = 0$  and  $z = a$ , respectively. The resulting line element for a non-rotating system is therefore [7]

$$ds^2 = -c^2 \left(1 + \epsilon \frac{z}{a}\right) dt^2 + dx^2 + dy^2 + dz^2 + O(|x|^2) = \eta_{\mu\nu} dx^\mu dx^\nu - \epsilon \frac{z}{a} c^2 dt^2,$$

where  $\eta_{\mu\nu}$  is the flat Minkowski metric  $\text{diag}(-1, 1, 1, 1)$ .

## 2. The energy–momentum tensor and the point-splitting procedure

In the point-splitting procedure, the energy–momentum tensor

$$T^{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}}$$

is obtained by introducing an auxiliary quantity  $\langle T^{\mu\nu'}(x, x') \rangle$  which involves the action of a differential operator on the Hadamard function [9, 10]. In the coincidence limit

$$\langle T^{\mu\nu}(x) \rangle = \lim_{x' \rightarrow x} \langle T^{\mu\nu'}(x, x') \rangle,$$

$\langle T^{\mu\nu}(x) \rangle$  is worked out. For QED (we use the Lorenz gauge [11] to obtain the standard wave operator on the potential)

$$S[A_\mu, \chi, \psi] = \int \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (\nabla^\mu A_\mu)^2 + \chi^{;\alpha} \psi_{;\alpha} \right] \sqrt{-g} d^4x,$$

one gets

$$\langle T^{\mu\nu} \rangle = \langle T_A^{\mu\nu} \rangle + \langle T_B^{\mu\nu} \rangle + \langle T_{\text{gh}}^{\mu\nu} \rangle, \tag{1}$$

with

$$\langle F_{\rho\alpha} F_{\tau\beta} \rangle = \lim_{x' \rightarrow x} \frac{1}{4} [H_{\alpha\beta';\rho\tau'} + H_{\beta\alpha';\tau\rho'} - H_{\alpha\tau';\rho\beta'} - H_{\tau\alpha';\beta\rho'} - H_{\rho\beta';\alpha\tau'} - H_{\beta\rho';\tau\alpha'} + H_{\rho\tau';\alpha\beta'} + H_{\tau\rho';\beta\alpha'}], \quad (2)$$

$$\langle T_A^{\mu\nu} \rangle = \lim_{x' \rightarrow x} \left[ -\frac{1}{4} (g^{\mu\rho} g^{\nu\tau} - \frac{1}{4} g^{\mu\nu} g^{\tau\rho}) g^{\alpha\beta} \langle F_{\rho\alpha} F_{\tau\beta} \rangle \right] \quad (3)$$

$$\langle T_B^{\mu\nu} \rangle = \lim_{x' \rightarrow x} \left[ -\frac{1}{4} g^{\alpha\beta} (g^{\mu\rho} g^{\nu\tau} + g^{\mu\tau} g^{\nu\rho} - g^{\mu\nu} g^{\tau\rho}) (H_{\beta\tau';\alpha\rho} + H_{\tau\beta';\rho\alpha'}) + \frac{1}{8} g^{\alpha\beta} g^{\mu\nu} g^{\rho\tau} (H_{\beta\tau';\alpha\rho'} + H_{\tau\beta';\rho\alpha'}) \right] \quad (4)$$

$$\langle T_{\text{gh}}^{\mu\nu} \rangle = \lim_{x' \rightarrow x} \left[ -\frac{1}{4} (g^{\mu\alpha} g^{\nu\beta} + g^{\mu\beta} g^{\nu\alpha} - g^{\mu\nu} g^{\alpha\beta}) (H_{;\alpha\beta'} + H_{;\beta\alpha'}) \right] \quad (5)$$

having defined

$$\begin{aligned} H_{\mu\nu}(x, x') &\equiv \langle [A_\mu(x), A_\nu(x')]_+ \rangle \equiv H_{\mu\nu}', \\ H(x, x') &\equiv \langle [\chi(x), \psi(x')]_+ \rangle, \\ [A_{\alpha;\rho}, A_{\beta;\tau}]_+ &\equiv \lim_{x' \rightarrow x} \frac{1}{2} \{ [A_{\alpha';\rho'}, A_{\beta;\tau}]_+ + [A_{\alpha;\rho}, A_{\beta';\tau'}]_+ \}. \end{aligned}$$

Since we need a recursive algorithm for the evaluation of Green functions, it is more convenient to work with the Feynman Green function instead of the Hadamard Green function. They are related through

$$H(x, x') = -2i[G(x, x') - \bar{G}(x, x')]$$

where  $\bar{G}(x, x') = \frac{1}{2}[G^+ + G^-]$ . The photon Green function  $G_{\lambda\nu'}$  in a curved spacetime with metric  $g_{\mu\nu}$  solves an equation of the form [12] ( $g_{\mu\nu'}$  being the parallel displacement bivector)

$$\sqrt{-g} P_\mu^\lambda(x) G_{\lambda\nu'} = g_{\mu\nu'} \delta(x, x') = g_{\mu\nu} \delta(x, x').$$

On expanding (this is, in general, only an asymptotic expansion)

$$G_{\lambda\nu'} \sim G_{\lambda\nu'}^{(0)} + \epsilon G_{\lambda\nu'}^{(1)} + O(\epsilon^2),$$

we get, to first order in  $\epsilon$ ,

$$\square^0 G_{\mu\nu'}^{(0)} = J_{\mu\nu'}^{(0)}, \quad (6)$$

$$\square^0 G_{\mu\nu'}^{(1)} = J_{\mu\nu'}^{(1)}, \quad (7)$$

where

$$J_{\mu\nu'}^{(0)} \equiv -\eta_{\mu\nu} \delta(x, x'),$$

$$\epsilon J_{\mu\nu'}^{(1)} \equiv \frac{z}{a} \epsilon \left( \frac{\eta_{\mu\nu}}{2} + \delta_\mu^0 \delta_\nu^0 \right) \delta(x, x') + 2\eta^{\rho\sigma} \Gamma_{\sigma\mu}^\tau G_{\tau\nu',\rho}^{(0)} + \eta^{\rho\sigma} \Gamma_{\rho\sigma}^\tau G_{\mu\nu',\tau}^{(0)} - \frac{z}{a} \epsilon G_{\mu\nu',00}^{(0)}$$

$$\text{with } \square^0 \equiv \eta^{\alpha\beta} \partial_\alpha \partial_\beta = -\partial_0^2 + \partial_x^2 + \partial_y^2 + \partial_z^2.$$

To fix the boundary conditions we note that, on denoting by  $\vec{E}_t$  and  $\vec{H}_n$  the tangential and normal components of the electric and magnetic fields, respectively, a sufficient condition to obtain

$$\vec{E}_t|_S = 0, \quad \vec{H}_n|_S = 0,$$

on the boundary  $S$  of the device, is to impose Dirichlet boundary conditions on [13]

$$A_0(\vec{x}), \quad A_1(\vec{x}), \quad A_2(\vec{x})$$

at the boundary  $z = 0, z = a$ . The boundary condition on  $A_3$  is determined by requiring that the gauge-fixing functional, here chosen to be of the Lorenz type, should vanish on the boundary (the boundary conditions on all components of  $A_\mu$  are then all preserved under gauge transformations [13] provided the same boundary condition on ghost fields is imposed, i.e. homogeneous Dirichlet). This implies

$$A_{;\mu}^\mu|_S = 0 \Rightarrow A_{;3}^3|_S = (g^{33}\partial_3 A_3 - g^{\mu\nu}\Gamma_{\mu\nu}^3 A_3)|_S = 0.$$

To first order in  $\epsilon$ , these conditions imply for Green functions the following:

$$\begin{aligned} G_{\mu\nu'}^{(0)}|_S &= 0, & \partial_3 G_{3\nu'}^{(0)}|_S &= 0, & \mu &= 0, 1, 2, \forall \nu', \\ G_{\mu\nu'}^{(1)}|_S &= 0, & \partial_3 G_{3\nu'}^{(1)}|_S &= -\frac{1}{2a} G_{3\nu'}^{(0)}|_S, & \mu &= 0, 1, 2, \forall \nu'. \end{aligned}$$

Hence we find that the third component of the potential  $A_\mu$  satisfies homogeneous Neumann boundary conditions to zeroth order in  $\epsilon$  and inhomogeneous boundary conditions to first order.

Now we are in a position to evaluate, at least formally (see below), the solutions to zeroth and first orders, and we get

$$G_{\lambda\nu'}^{(0)} = \eta_{\lambda\nu'} \int \frac{d\omega d^2k}{(2\pi)^3} e^{-i\omega(t-t') + i\vec{k}_\perp \cdot (\vec{x}_\perp - \vec{x}'_\perp)} [(1 - \delta_{\lambda 3})g_D(z, z') + \delta_{\lambda 3}g_N(z, z')], \quad (8)$$

having defined

$$\begin{aligned} g_D(z, z'; \kappa) &\equiv \frac{\sin \kappa(z_<) \sin \kappa(a - z_>)}{\kappa \sin \kappa a}, & 0 < z, z' < a, \\ g_N(z, z'; \kappa) &\equiv -\frac{\cos \kappa(z_<) \cos \kappa(a - z_>)}{\kappa \sin \kappa a}, & 0 < z, z' < a, \end{aligned}$$

where  $D, N$  stand for homogeneous Dirichlet or Neumann boundary conditions, respectively,  $z_> (z_<)$  are the larger (smaller) between  $z$  and  $z'$ , while  $\vec{k}_\perp$  has components  $(k_x, k_y)$ ,  $\vec{x}_\perp$  has components  $(x, y)$ ,  $\kappa \equiv \sqrt{\omega^2 - k^2}$  and

$$G_{\mu\nu'}^{(1)} = \int \frac{d\omega d^2k}{(2\pi)^3} e^{-i\omega(t-t') + i\vec{k}_\perp \cdot (\vec{x}_\perp - \vec{x}'_\perp)} \Phi_{\mu\nu'}, \quad (9)$$

where the  $\Phi$  components different from zero are written in [14]. The ghost field satisfies the same equations as the 22 component of the gauge field; hence we do not write it explicitly. In the following we write simply  $G_{\mu\nu'}$  and  $G$  for the Green function of the gauge and ghost field, respectively.

We should stress at this stage that, in general, the integrals defining the Green functions are divergent. They are well defined as long as  $x \neq x'$ ; hence we will perform all our calculations maintaining the points separated and only in the very end shall we take the coincidence limit as  $x' \rightarrow x$  [15]. We have decided to write the divergent terms explicitly so as to bear them in mind and remove them only in the final calculations by hand, instead of making the subtraction at an earlier stage.

Our Green functions are found to satisfy the Ward identity

$$G_{\nu';\mu}^\mu + G_{;\nu'} = 0, \quad G_{\nu'}^{\mu;\nu'} + G^{;\mu} = 0,$$

to first order in  $\epsilon$  so that, to this order, gauge invariance is explicitly preserved. Ward identities imply  $\langle T_B^{\mu\nu} \rangle + \langle T_{gh}^{\mu\nu} \rangle = 0$  to first order in  $\epsilon$ , thus in the following we do not consider them. Nonetheless we explicitly computed them and verified that they cancel each other.

### 3. Energy–momentum tensor

Using equations (1)–(3) we get, from the asymptotic expansion  $T_{\mu\nu'} \sim T_{\mu\nu'}^{(0)} + \frac{\epsilon}{a} T_{\mu\nu'}^{(1)} + O(\epsilon^2)$ ,

$$\langle T^{(0)\mu\nu'} \rangle = \frac{1}{16a^4\pi^2} \left( \zeta_H \left( 4, \frac{2a+z-z'}{2a} \right) + \zeta_H \left( 4, \frac{z'-z}{2a} \right) \right) \text{diag}(-1, 1, 1, -3), \quad (10)$$

where  $\zeta_H$  is the Hurwitz  $\zeta$ -function  $\zeta_H(x, \beta) \equiv \sum_{n=0}^{\infty} (n + \beta)^{-x}$ . On taking the limit  $z' \rightarrow z^+$  we find

$$\lim_{z' \rightarrow z^+} \langle T^{(0)\mu\nu'} \rangle = \left( \frac{\pi^2}{720a^4} + \lim_{z' \rightarrow z^+} \frac{1}{\pi^2(z-z')^4} \right) \text{diag}(-1, 1, 1, -3), \quad (11)$$

where the divergent term as  $z' \rightarrow z$  can be removed by subtracting the contribution of infinite space without bounding surfaces [1], and in our analysis we therefore discard it hereafter. The renormalization of the energy–momentum tensor in the curved spacetime is usually performed by subtracting the  $\langle T_{\mu\nu} \rangle$  constructed with an Hadamard or Schwinger–DeWitt two-point function up to the fourth adiabatic order [9, 16]. In our problem, however, as we work to first order in  $\epsilon$ , we neglect tidal forces and therefore the geometry of spacetime in between the plates is flat. Thus, we need only subtract the contribution to the energy–momentum tensor that is independent of  $a$ , which is the standard subtraction in the context of the Casimir effect in flat spacetime [17].

In the same way we get, to first order in  $\epsilon$ :

$$\lim_{z' \rightarrow z^+} \langle T^{(1)\mu\nu'} \rangle = \text{diag}(T^{(1)00}, T^{(1)11}, T^{(1)22}, T^{(1)33}) + \lim_{z' \rightarrow z^+} \text{diag}(-z'/\pi^2(z-z')^4, 0, 0, 0), \quad (12)$$

where

$$T^{(1)00} = -\frac{\pi^2}{1200a^3} + \frac{\pi^2 z}{3600a^4} + \frac{\pi}{240a^3} \frac{\cos\left(\frac{\pi z}{a}\right)}{\sin^3\left(\frac{\pi z}{a}\right)}, \quad (13)$$

$$T^{(1)11} = \frac{\pi^2}{3600a^3} - \frac{\pi^2 z}{1800a^4} - \frac{\pi}{120a^3} \frac{\cos\left(\frac{\pi z}{a}\right)}{\sin^3\left(\frac{\pi z}{a}\right)}, \quad (14)$$

$$T^{(1)22} = T^{(1)11}, \quad (15)$$

$$T^{(1)33} = -\frac{(\pi^2(a-2z))}{720a^4}. \quad (16)$$

Incidentally, we note that the tensor is covariantly conserved:  $\nabla \cdot T = 0$  to first order in  $\epsilon$ .

### 4. Push and trace anomaly

To compute the Casimir energy we must project the energy–momentum tensor along the unit time-like vector  $u$  with covariant components  $u_\mu = (\sqrt{-g_{00}}, 0, 0, 0)$  to obtain  $\rho = \langle T^{\mu\nu} \rangle u_\mu u_\nu$ , so that

$$\rho = -\frac{\pi^2}{720a^4} + 2\frac{g}{c^2} \left( -\frac{\pi^2}{1200a^3} - \frac{\pi^2 z}{900a^4} + \frac{\pi}{240a^3} \frac{\cos\left(\frac{\pi z}{a}\right)}{\sin^3\left(\frac{\pi z}{a}\right)} \right) + O(g^2),$$

where in the second line we have replaced  $\epsilon$  by its expression in terms of  $g$ . Thus, the energy stored in the Casimir device is found to be

$$E = \int d^3\Sigma \sqrt{-g} \langle T^{\mu\nu} \rangle u_\mu u_\nu = -\frac{\hbar c \pi^2}{720} \frac{A}{a^3} \left( 1 + \frac{5ga}{2c^2} \right) \equiv E_C \left( 1 + \frac{5ga}{2c^2} \right),$$

where  $A$  is the area of the plates,  $d^3\Sigma$  is the 3-volume element of an observer with 4-velocity  $u_\mu$ , the integral has been computed as a principal-value integral, and we have reintroduced  $\hbar$  and  $c$ .

In the same way, the pressure on the plates is given by

$$P(z=0) = \frac{\pi^2 \hbar c}{240 a^4} \left(1 + \frac{2ga}{3c^2}\right), \quad P(z=a) = -\frac{\pi^2 \hbar c}{240 a^4} \left(1 - \frac{2ga}{3c^2}\right).$$

To obtain the resulting force one has to multiply each of them by the redshift  $r$  of the point where they act, relative to the point where they are added [18]:

$$r_{P_{\text{added}}(P_{\text{act}})} = \sqrt{\frac{|g_{00}(P_{\text{act}})|}{|g_{00}(P_{\text{added}})|}} \approx 1 + \frac{g}{c^2}(z - z_Q)$$

to leading order in  $\frac{gz}{c^2}$ , so that a net force [19]

$$F = -\frac{\pi^2 \hbar c}{a^4} \left[ \frac{g}{240c^2}(z_2 - z_1) - \frac{4g}{720c^2}(z_2 - z_1) \right] = \frac{\pi^2 A \hbar g}{720 ca^3} = \frac{E_C}{c^2} g,$$

pointing upwards along the  $z$ -axis is obtained, in perfect agreement with the early findings of [20] and the more recent results of [21].

From the previous expressions of the energy–momentum tensor the following trace anomaly  $\tau$  is obtained:

$$\tau = \frac{\hbar g}{ca^3} \left( \frac{\pi^2 z}{180a} - \frac{\pi \cos\left(\frac{\pi z}{a}\right)}{24 \sin^3\left(\frac{\pi z}{a}\right)} \right).$$

The volume integral of this density exists as a principal-value integral and is given by

$$\int \tau d^3\Sigma = \frac{\pi^2 \hbar g}{360 ca^2} A.$$

The global integrated form of the trace anomaly is the new result with respect to the analysis in [6]. It tends to zero at a large separation  $a$  between the plates. This trace anomaly is therefore caused by the presence of the boundaries and thus is of a different nature from the usual trace anomaly which is encountered in curved spacetimes without boundaries, which depends on local invariants built from the Riemann curvature [15, 16, 22].

## 5. Concluding remarks

To the best of our knowledge, the analysis presented in this paper represents the first study of the energy–momentum tensor for the electromagnetic field in a Casimir cavity placed in a weak gravitational field. The resulting calculations are considerably harder than in the case of scalar fields. By using Green-function techniques, we have evaluated the influence of the gravity acceleration on the regularized energy–momentum tensor of the quantized electromagnetic field between two plane-parallel ideal metallic plates, at rest in the gravitational field of the earth, and lying in a horizontal plane. In particular, we have obtained a detailed derivation of the theoretical prediction according to which a Casimir device in a weak gravitational field will experience a small push in the upward direction [6]. This result is consistent with the picture that the *negative* Casimir energy in a gravitational field will behave like a *negative mass*. Furthermore, we find a trace anomaly proportional to the gravitational acceleration and vanishing for infinite plate separation, not previously worked out for a Casimir device in a gravitational field. An outstanding open problem is now how to obtain an independent demonstration of our formula for the trace anomaly, and what its implications are for fundamental physics.

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